

# Numerical Analysis of Eddy Current Non Destructive Testing (JSAEM Benchmark Problem #6- Cracks with Different Shapes)

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## Abstract

In this paper we show some numerical calculation of the sixth JSAEM - E'NDE benchmark problem. The problem treats the Eddy Current Testing signals of four test pieces with different shape made by electrical discharge machining notches. We have solved the problems by means of a Finite Elements approach. In the paper we give the main features of this numerical approach, and some information about the hardware board used and about the needs of memory allocation and of CPU time of the software used. The numerical calculated values are compared with measurements.

## 1 Introduction

One of the activities in the research field about the electromagnetic non destructive evaluation is the discussion dealing with the efficiency of different numerical methods in solving both forward and inverse problems. Recently six benchmark problems have been discussed and proposed at the Research Committee on Advanced Eddy Current Testing Technology of the Japan Society of Applied Electromagnetics and Mechanics [1] [2] [3]. We continue our activity of analysis and numerical evaluation of these benchmark problems [4] [5] and we present here the results obtained for the problem #6 – Cracks with different shapes.

## 2 Brief Description of the Problem

The problems #6 deals with a pancake type coil, placed above a flat plate with a crack. The probe coil moves parallel to the x-axis (see fig. 1a), placed along the crack length direction. The probe coil has axis-symmetric shape and is made of 140 turns (see fig 1b). It is supplied with a current of an amplitude of 8mA (rms value). The plate is made of Inconel 600 and has an electric conductivity  $\sigma = 1 \cdot 10^6 \Omega^{-1} m^{-1}$  and a relative magnetic permeability  $\mu_r = 1.0$ .

The solution of the forward problem requires the determination of the impedance change of the probe. This parameter is evaluated by subtracting the values obtained for the plate without crack from the values obtained for the plate with crack. The impedance change must be calculated at the frequency of 150 kHz or of 300 kHz and for lift-off of 0.5mm at different coil locations, starting from  $x = -10$  mm till  $x = 10$  mm at every 1 mm along the crack direction. Range of measurement is  $-10 \leq x \leq 10$  (mm),  $-5 \leq y \leq 5$  (mm), every 1mm.

The solution of the inverse problem deals with reconstruction of the shapes of notches.

Four different types of crack having the following shapes and length of 10 mm are taken in account (see fig. 2):

- (a) Elliptical shape crack width 0.22 mm
- (b) Stepwise shape crack width 0.25 mm
- (c) Slope shape crack width 0.23 mm
- (d) Rectangular shape crack width 0.22 mm

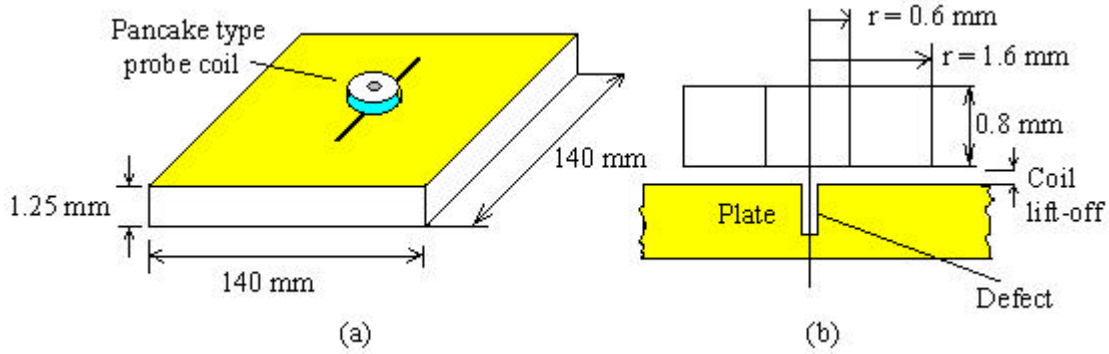


Fig. 1 - Description of the benchmark geometry

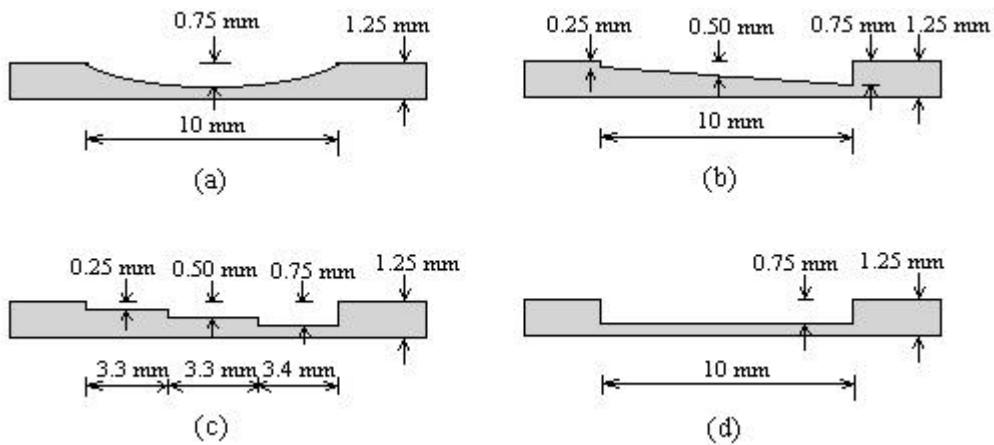


Fig. 2 - Description of the benchmark geometry

### 3 Numerical Approach

Finite element methods using linear tetrahedron edge elements have been developed to compute three-dimensional distributions of the eddy current and magnetic flux density.

The sinusoidal time dependence between magnetic field  $\mathbf{H}$ , current density  $\mathbf{J}$ , electric flux density  $\mathbf{D}$  is given by

$$\nabla \times \mathbf{H} = j\omega \mathbf{D} + \mathbf{J} \quad (1)$$

where  $\omega$  is the angular frequency.

Using a vector weighting function  $N_i$  is possible to obtain the Galerkin's weighted residual equation from Eq.(1) as follows:

$$\iiint_{\Omega} \mathbf{N}_i \cdot (\nabla \times \mathbf{H} - j\omega \mathbf{D} - \mathbf{J}) d\Omega = 0 \quad (2)$$

Where  $\Omega$  is the domain model analysis. A magnetic vector potential  $\mathbf{A}$  is introduced as unknown vector variable.

Substituting:

$$\mathbf{H} = \frac{\mathbf{B}}{m} \quad (3)$$

with

$$\mathbf{B} = \nabla \times \mathbf{A} \quad (4)$$

and

$$\mathbf{D} = \mathbf{e}^* \mathbf{E} \quad (5)$$

with

$$\mathbf{E} = -j\omega \mathbf{A} \quad (6)$$

where  $\mathbf{e}^* = \mathbf{e} + \frac{\mathbf{s}}{j\omega}$  is the complex permittivity. The boundary conditions used are either to impose  $\mathbf{A}=0$  or use infinite elements [6].

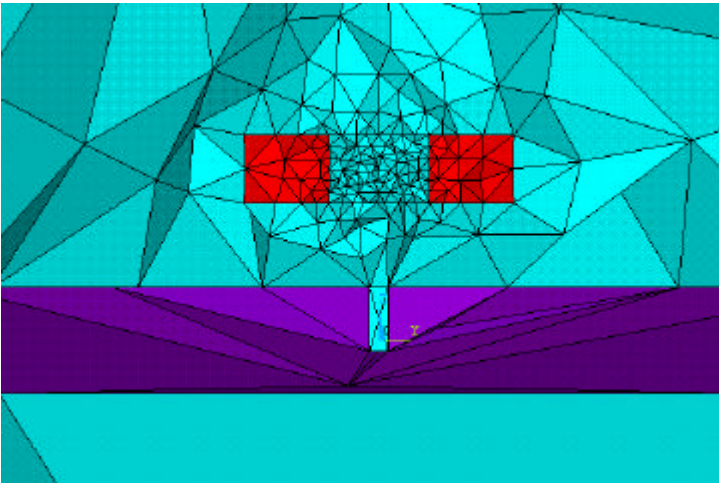
## 4 Results

We have used for the numerical calculation a PC with Pentium II INTEL<sup>®</sup> 350 MHz Processor with 256 MB RAM. The finite element numerical code used is ANSYS<sup>®</sup> Version 5.5. The values of the equivalent impedance of the system have been calculated by means of the circuitual mixed block implemented in ANSYS<sup>®</sup> using the element named CIRCU 124. Complete computation typical times are 150 minutes for each simulation. In table I are reported the details about the numerical method used.

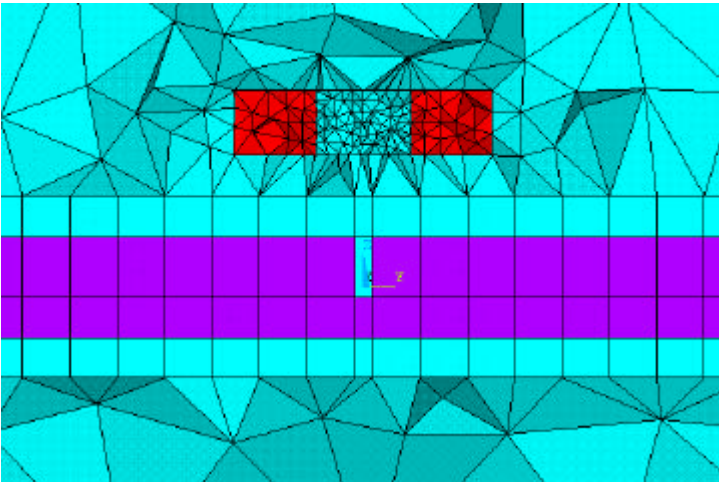
In fig. 3 a, b and c are reported three different meshes used for the modelling of the geometry. It can be noted that the pre-processor, using mapped meshes allows more regular subdivision of the plate, and this fact determines a more accurate simulation of the induced current flow around the cracks. For clarity the mesh representations in these figures are two dimensional. Fig. 4 show the mesh in three dimensional. In fig. 5 is shown the typical trend of the degree of accuracy obtained with the different meshes techniques and with the increasing of the discretization.

In our numerical experiments the calculated values obtained using the mesh type c were always the closer ones to the measurements. Moreover it can be seen by the fig. 6 and 7 that the distribution of  $\bar{Z}$  (either real or imaginary part) versus the position of the coil respect to the defect are always of the same shape of those measured. The accuracy of the calculation is in general satisfactory, even if to obtain good results a quite large calculation time has necessary. Finally we have seen that the measured values of the impedance are always higher then the computed ones.

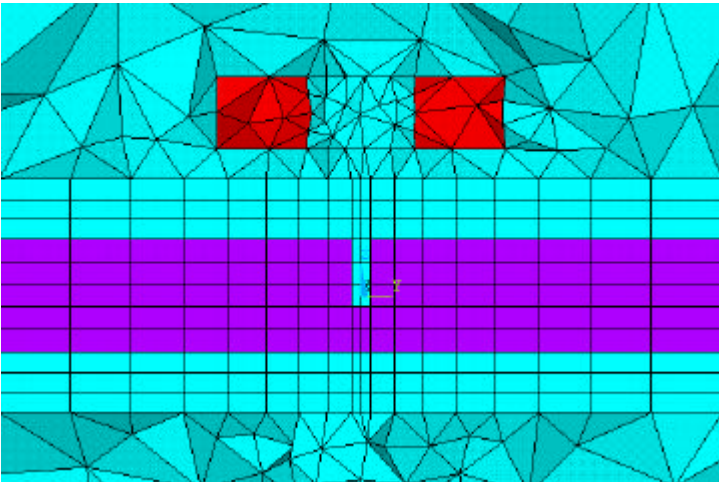
This fact confirms previous results obtained by other researchers, again using FEM codes [7]. In fig.6 and 7 are summarised some calculated results. In the same figure are pictured the measured values [8].



(a) full free mesh with tetrahedral shape element, 2995 nodes, 2335 elements



(b) free mesh with tetrahedral shape element and mapped mesh with hexahedral shape element, 9314 nodes, 7472 elements



(c) free mesh with tetrahedral shape element and mapped mesh with hexahedral shape element, 18852 nodes, 15181 elements

Fig. 3 – Meshes for the plate with crack and for the probe coil

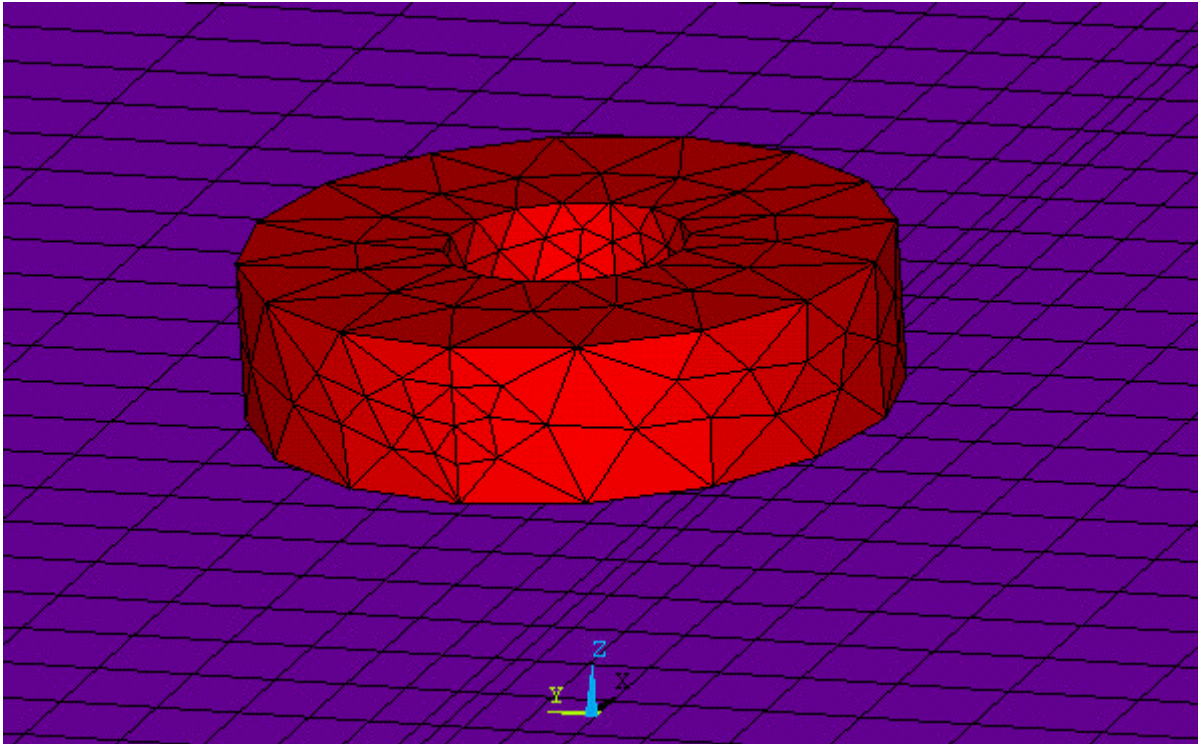


Fig. 4 – Meshes for the plate with crack and for the probe coil three dimensional

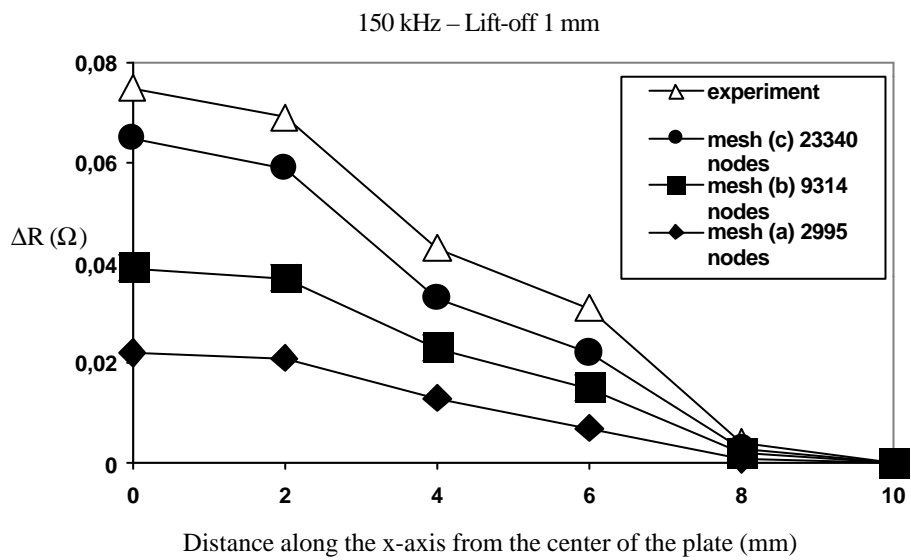


Fig. 5 – Results of the different meshes

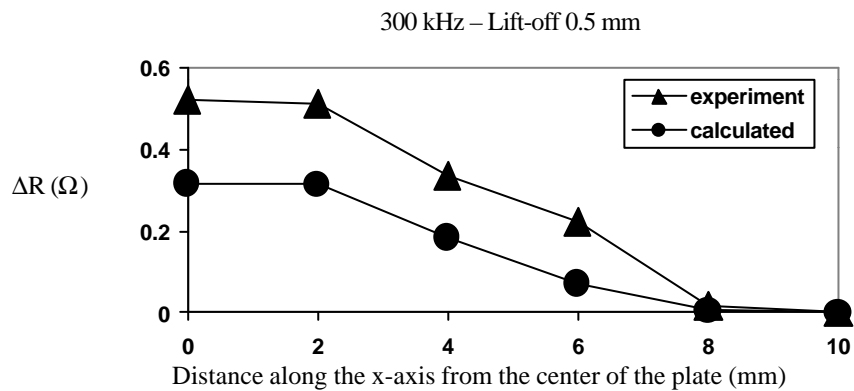
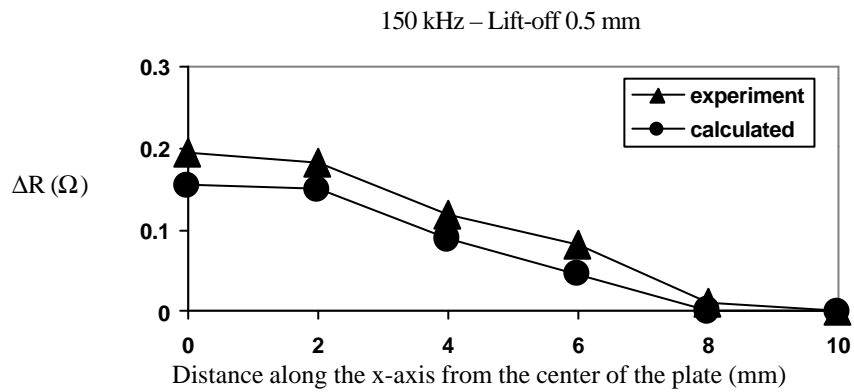
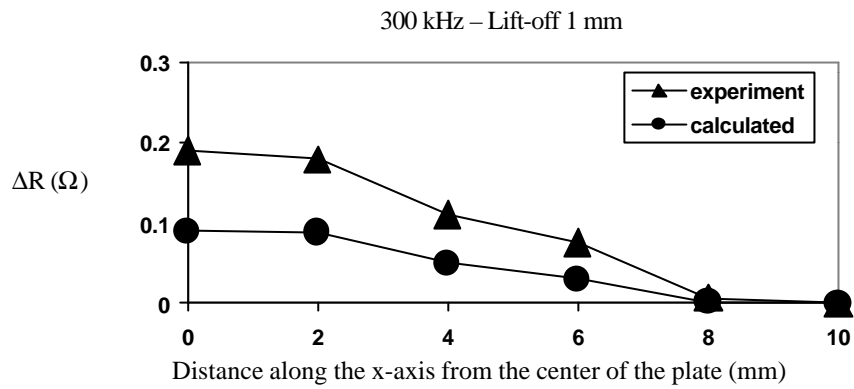
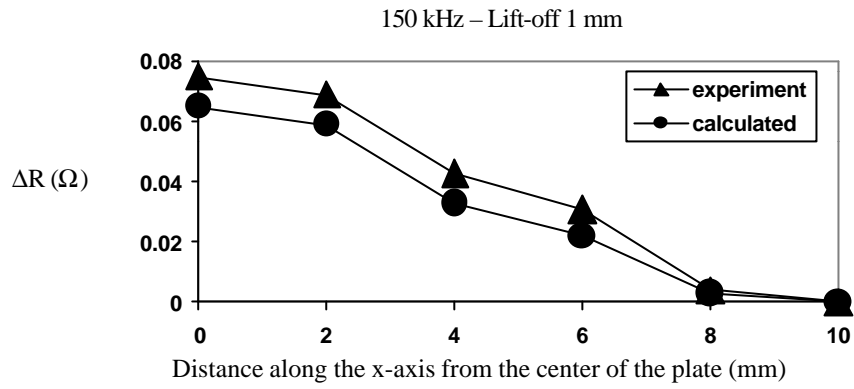


Fig. 6 – Resistance changes for the rectangular crack

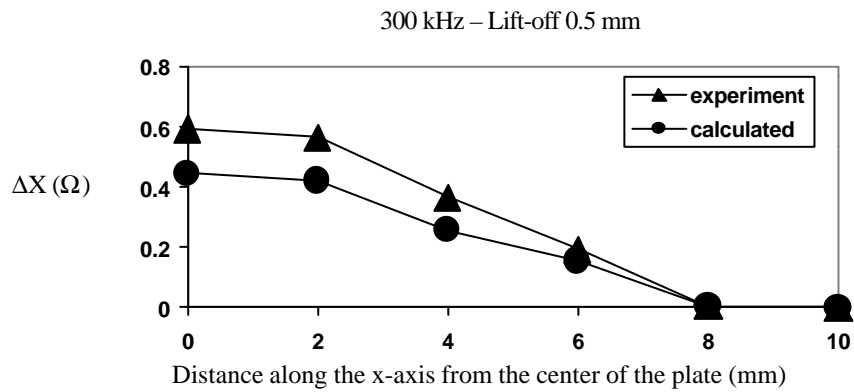
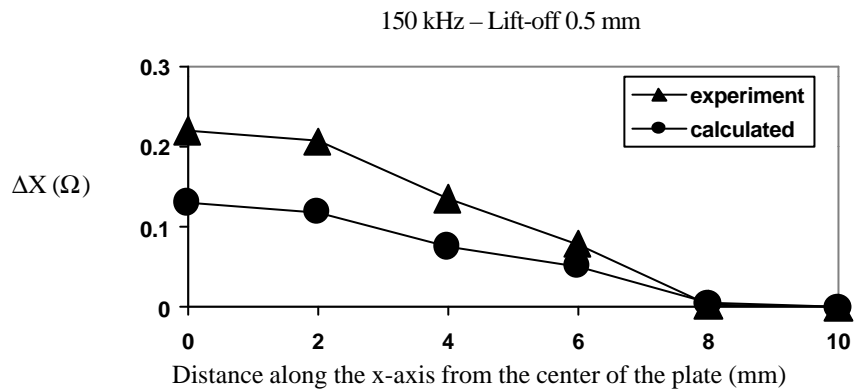
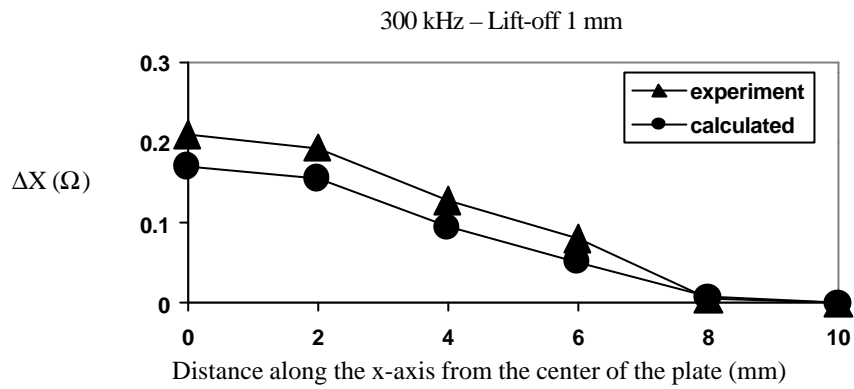
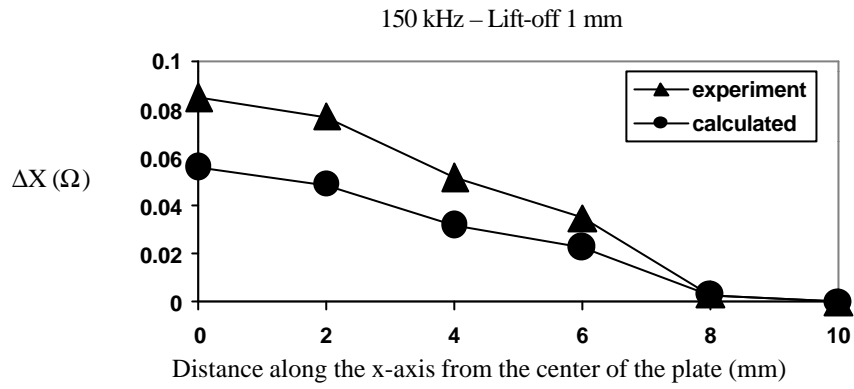


Fig. 7 – Reactance changes for the rectangular crack

**Table 1**

**Description of computer program**

No.	Item	Specification
1	Code name	ANSYS theory Reference 001099 9 <sup>th</sup> Editions. SAS IP, Inc. ©
2	Formulation	4. FEM (Finite Element Method)

3	Governing equations	In conductor	$\nabla \times \frac{1}{\mathbf{m}} \nabla \times \dot{\mathbf{A}} + j\omega \mathbf{s} \dot{\mathbf{A}} + \mathbf{s} \nabla \dot{V} - \frac{\mathbf{s}}{\mathbf{m}} \nabla \times \dot{\mathbf{A}} = 0$ $\nabla \cdot \left( -j\omega \mathbf{s} \dot{\mathbf{A}} - \mathbf{s} \nabla \dot{V} + \frac{\mathbf{s}}{\mathbf{m}} \nabla \times \dot{\mathbf{A}} \right) = 0$
		In vacuum	$\nabla \times \frac{1}{\mathbf{m}} \nabla \times \dot{\mathbf{A}} = \mathbf{J}_s$
4	Solution variables	In conductor	$\dot{\mathbf{A}}, \dot{V}$
		In vacuum	$\dot{\mathbf{A}}$

5	Gauge condition	Coulomb Gauge
6	Treatment of exciting current	Element SOLID 97 + Applied Current Loads

7	Impedance calculation method	Equation	$\bar{Z} = \frac{\dot{V}_2 - \dot{V}_1}{i}$
		Description	

8	Solution method for equation	Newton-Raphson
	Convergence criterion for interaction method	Max relative error < 10 <sup>-3</sup>
9	Element types	SOLID 97, CIRCU 124
10	Number of elements	15181
11	Number of nodes	18852
12	Number of unknowns	45543

13	Computer	Name	PC Intel Pentium II 350 MHz
		Speed	128 MIPS 15 MFLOPS
		Main memory (MB)	256
		Used Memory (MB)	60



	Precision of data (bits)	32
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	CPU time (s)	total	9000
		solving linear equations	8100

14	References on computer program	Ansys Reference Manual Release 5.5
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## References

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